Series Q5QPS/5

Set - 2

प्रश्न-पत्र कोड
Q.P. Code **65/5/2**

अनुक्रमांक
Roll No.

2 6 6 [redacted]

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* — *अधिकतम अंक : **80***

*Time allowed : **3 hours*** — *Maximum Marks : **80***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A**, **B**, C, **D** and **E**.*

(iii) *In **Section A** – Questions Number **1** to **18** are Multiple Choice Questions (MCQs) type and Questions Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Questions Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Questions Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Questions Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Questions Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculators is **NOT** allowed.*

SECTION - A

This section has **20** multiple choice questions of **1** mark each.

1. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

   (A) $\sin x \, e^{\sin^2 x}$ (B) $\cos x \, e^{\sin^2 x}$

   (C) $-2 \cos x \, e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x \, e^{\sin^2 x}$

2. If A is a square matrix of order 2 and $|A| = -2$, then value of $|5A'|$ is :

   (A) -50 (B) -10

   (C) 10 (D) 50

3. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

   (A) 2 (B) 1

   (C) 0 (D) -2

4. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

   (A) -60 units/sec (B) 60 units/sec

   (C) -70 units/sec (D) -140 units/sec

5. The product of matrix P and Q is equal to a diagonal matrix. If the order of matrix Q is 3×2, then order of matrix P is :

   (A) 2×2 (B) 3×3

   (C) 2×3 (D) 3×2

6. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

   (A) injective but not surjective. (B) surjective but not injective.

   (C) both injective and surjective. (D) neither injective nor surjective.

7. If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to :

(A) $\frac{x}{y}$

(B) $-\frac{x}{y}$

(C) $\frac{y}{x}$

(D) $-\frac{y}{x}$

8. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

(A) -4

(B) 1

(C) 3

(D) 4

9. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j) :

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

10. If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is :

(A) $\sqrt{2}$

(B) $-\sqrt{2}$

(C) 8

(D) $2\sqrt{2}$

11. The value of $\int_{\pi/4}^{\pi/2} \cot\theta \, \text{cosec}^2\theta \, d\theta$ is :

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) 0

(D) $-\frac{\pi}{8}$

12. The integral $\int \frac{dx}{\sqrt{9-4x^2}}$ is equal to :

(A) $\frac{1}{6}\sin^{-1}\left(\frac{2x}{3}\right)+c$

(B) $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$

(C) $\sin^{-1}\left(\frac{2x}{3}\right)+c$

(D) $\frac{3}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$

13. The area of the region bounded by the curve $y^2 = 4x$ and $x = 1$ is :

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) $\frac{64}{3}$

(D) $\frac{32}{3}$

14. The general solution of the differential equation

$\frac{dy}{dx} = e^{x+y}$ is :

(A) $e^x + e^{-y} = c$

(B) $e^{-x} + e^{-y} = c$

(C) $e^{x+y} = c$

(D) $2e^{x+y} = c$

15. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(A) $\frac{5\pi}{6}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$

(D) $\frac{7\pi}{4}$

16. The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line :

$\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

17. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :

(A) $A \subset B$, but $A \neq B$ (B) $A = B$

(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

18. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\dfrac{\vec{p} + 3\vec{q}}{4}$ (B) $\dfrac{\vec{p} + 3\vec{q}}{8}$

(C) $\dfrac{5\vec{p} + 3\vec{q}}{4}$ (D) $\dfrac{5\vec{p} + 3\vec{q}}{8}$

Assertion – Reason Based Questions

Direction : In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The vectors

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

20. **Assertion (A)** : Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

SECTION - B

This section has **5** Very Short Answer questions of **2** marks each.

21. If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$ and

$b = \tan^{-1}\left(\sqrt{3}\right) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

then find the value of a + b.

22. (a) Find : $\int \cos^3 x \, e^{\log \sin x} \, dx$

OR

(b) Find : $\int \frac{1}{5 + 4x - x^2} \, dx$

23. Sand is pouring from a pipe at the rate of 15 cm^3/minute. The falling sand forms a cone on the ground such that the height of the cone is always one-third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4 cm ?

24. Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes.

25. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

SECTION – C

There are 6 short answer questions in this section. Each is of 3 marks.

26. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2} ; y(0) = 5.$$

OR

(b) Solve the following differential equation :

$$x^2 \, dy + y(x + y) \, dx = 0$$

27. Find the values of a and b so that the following function is differentiable for all values of x :

$$f(x) = \begin{cases} ax + b , & x > -1 \\ bx^2 - 3 , & x \leq -1 \end{cases}$$

28. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

29. (a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

(b) Find : $\int \frac{2x+1}{(x+1)^2 (x-1)} dx$

30. Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 3$.

31. Bag I contains 3 red and 4 black balls. Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

SECTION - D

There are **4** long answer questions in this section. Each question is of **5** marks.

32. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through (2, –1, 1) and parallel to $\frac{x}{1}=\frac{y}{1}=\frac{z}{3}$

$L_2 : \vec{r}=\hat{i}+(2\mu+1)\hat{j}-(\mu+2)\hat{k}$.

33. (a) If $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 0 & -3\\ 1 & 2 & 0\end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$$x+2y-3z=1$$
$$2x-3z=2$$
$$x+2y=3$$

OR

(b) Find the product of the matrices $\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{bmatrix}$ and hence solve the system of linear equations :

$$x+2y-3z=-4$$
$$2x+3y+2z=2$$
$$3x-3y-4z=11$$

34. Find the area of the region bounded by the curve $4x^2+y^2=36$ using integration.

35. Solve the following Linear Programming problem graphically :

Maximise $Z = 300x + 600y$

Subject to $x + 2y \leq 12$

$2x + y \leq 12$

$x + \frac{5}{4}y \geq 5$

$x \geq 0, y \geq 0.$

SECTION – E

In this section there are **3** case study questions of **4** marks each.

36. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.

Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

37. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

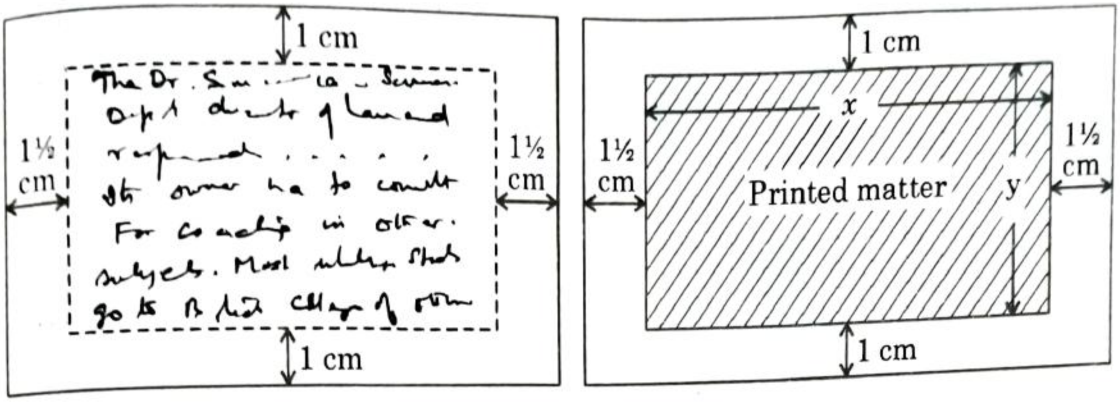
(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

38. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.